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MULTICRITERIA EVALUATION OF LOT SIZING TECHNIQUES AS A FUNCTION OF DEMAND PATTERN, TIME BETWEEN ORDERS, AND DEMAND VARIABILITY

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The interactive effects of demand pattern (classified according to forecasting difficulty), cost structure (represented by time between orders), and demand variability on the performance of five lot sizing techniques (Eisenhut Part Period Balance, EOQ, Silver-Meal, Tsado, and Wagner-Whitin heuristic) were evaluated using empirical demand data from local businesses. Three performance criteria were used: inventory cost relative to the Wagner-Whitin optimal, number of stockouts, and percentage short per stockout. The results indicate that selecting the best algorithm depends on the performance criterion selected. EOQ, Tsado, and Silver-Meal performed best with respect to relative inventory cost, while Eisenhut and Wagner-Whitin heuristic performed best with respect to the shortage criteria. Percentage short per stockout was affected by differences in forecasting difficulty. The effect was constant across all techniques. Significant differences among all three classes of demand variability were found for all three criteria. Level of forecasting difficulty and variability do not affect the optimal choice of lot sizing technique if focus forecasting is used.

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This research report entitled "Multicriteria Evaluation of Lot Sizing Techniques As A Function of Demand Pattern, Time Between Orders, and Demand Variability" is presented as a competent treatment of the subject, worthy of publication. The United States Air Force Academy vouches for the quality of the research, without necessarily endorsing the opinions and conclusions of the author.

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MULTICRITERIA EVALUATION OF LOT SIZING TECHNIQUES AS A FUNCTION OF DEMAND PATTERN, TIME BETWEEN ORDERS, AND DEMAND VARIABILITY

By Bryan S. Cline, Bobbie L. Foote, and Robert E. Schlegel

INTRODUCTION

The motivation for this research was the observation of procurement systems at military logistics agencies. The inventory ordering systems at these facilities are automated and long years of experience with these systems are available. The data result in patterns of forecasting error which sometimes favor one logistics approach over another. Level demand sets in which the mean is constant cause no problems for (R, Q) models with exponential smoothing as the forecasting approach. However, if the demand pattern exhibits trends or has many level changes, forecasting is difficult and the fluctuations in R and Q that arise are very expensive. In the latter case, buying 9 or 12 months demand plus a buffer stock based on a forecast at a given time of the year may be much more effective in terms of both service level and inventory cost. Thus, a change in logistics approach is dictated by the demand pattern forecasting difficulty (Foote, Kebriaei and Kuman, 1988).

The criteria used in inventory procurement is also important in reducing the harmful effects of demand variance. For example, if shortages are recognized explicitely by modeling shortage costs, changes in demand variance have a big impact on order quantity. If shortage is modeled by service level constraints, changes in variance have a big impact on triggering an order. Different lot sizing techniques will give different order quantities when the same forecast is used. Liberal lot sizing techniques that "over order" will protect against shortages while conservative techniques will reduce or eliminate holding costs.

It is also known that lot sizing tends to perform worse in practice than in simulated tests. The reason for this is that actual demand patterns change over time from constant at a particular level, to

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trending, to constant at a new level. A change to trending in the data triggers a false estimate of the variance when techniques such as exponential smoothing are used. Based on observations of actual demand pattern changes, a study was conducted to investigate the interaction of (1) lot sizing approach, (2) ratio of holding cost to setup cost (defining time between orders or "TBO"), (3) an implied shortage cost model criterion, and (4) the demand pattern based on actual demands obtained from a large national distribution company and a vendor for fast food chains. The selection of the forecasting techniques, lot sizing approaches, and cost factors was based on an extensive survey by Cline (1989).

FORECASTING TECHNIQUE

As discussed above, the accuracy of various forecasting techniques in terms of estimating demand variance, recognizing trending, and consistently estimating the mean is important in reducing inventory costs. Barron and Targett (1985), Flores and Whybark (1986), and Amar and Gupta (1986) all point out that forecasting methods perform differently on simulated data compared with empirical data. Based on the results of a forecasting competition, Makridakis et al. (1982) established the following principles for selecting a forecasting technique: (1) knowledge of the underlying demand pattern helps identify the best technique, (2) simple models work well, and (3) the average of the forecasts from simple models is superior to the forecast from a single model.

From the literature, exponential smoothing is more powerful compared with a large number of alternative approaches and is widely used as a baseline technique (Silver and Peterson, 1985). If trending in the data occurs, Holt's exponential smoothing model may be used to prevent the forecasting technique from becoming disadvantageous (Ibid.). Based on industrial short course projects conducted from 1987 to 1989, we used the idea of tracking the mean absolute deviation (MAD) and bias of a set of exponentially smoothed forecasts and selecting the best forecast for the next planning horizon. Silver and Peterson (1985) argue, however, that changing the smoothing constants (what they refer to as "adaptive" smoothing), while having considerable intuitive appeal, is "not necessarily better than regular, non-adaptive smoothing." (e.g., Ekern, 1981; Flowers, 1980; Gardner and Dannenbring, 1980). Specifically, Silver and Peterson feel the resulting forecasts would be excessively "nervous". However, it is our contention that if a forecast model is set, this effect does not occur. We also believe the lot sizing problem studied here only requires the use of an extended forecast approximately once every "TBO" period.

Since we used automated forecasting, the best model for the current data could not be easily determined. Thus, only the average of exponential models with varying parameters or "focus forecasting" which selects a parameter set for Holt's model based on MAD and bias could be used.

During program tests on samples of empirical data, the focus forecasting approach performed better than averaging techniques in terms of less deviation and bias although the difference was not statistically significant. Comparisons of the MAD for the focus procedure with the MADs for each individual, static procedure were quite favorable. Focus forecasting was selected for this study since it was intuitively more appealing and its history in terms of student forecasting competitions reflected great success. Rather than use a single model with changing parameters based on error, we used several models with different fixed parameter sets. When a new, extended forecast was needed, the model which had the best forecast history up to that point was chosen.

LOT SIZING TECHNIQUES

The following lot sizing techniques were compared: Wagner-Whitin (W-W), Eisenhut (Part Period Balancing or PPB), Silver-Meal, EOQ, and Tsado. The W-W approach (Silver and Peterson, 1985) has the advantage of claiming optimality over the time horizon for deterministic demand but requires zero demand after the last period (after the time horizon). Actual demand does not satisfy either assumption; however, Wemmerlov and Whybark (1984) indicate the W-W approach is one of the best procedures when simulated forecast errors are introduced.

Eisenhut part period balancing and EOQ do well in many cases when stochastic demand is introduced (Callarman and Hamrin, 1979). Silver-Meal (S-M) has performed well in some studies with forecast errors (De Bodt and Van Wassenhove, 1981) and poorly in others (Callarman and Hamrin, 1979). Using actual demand data with simulated error, the EOQ, S-M, PPB, and least cost lot sizing methods adjusted to integral periods of demand were indistinguishable (De Bodt and Van Wassenhove, 1983).

Tsado (1985a, b) developed a heuristic specifically to deal with stochastic demand and tested the method on historical data. Tsado used the entire demand history to produce the forecast and then computed lot sizes based on several approaches. His method appeared to work well compared with the W-W, EOQ, PPB, and De Bodt's modified Silver-Meal. Unfortunately, industry normally has only a limited demand history. As a result, Tsado's methodology cannot be typically used.

Wemmerlov (1989) analyzed the effect of data variance on the performance of 14 lot sizing methods using simulated data. He found that, without forecast errors, W-W was best, followed by Groff's rule and S-M. With errors, Eisenhut PPB was best, followed by two of Wemmerlov's rules. EOQ and W-W were in the top six.

PERFORMANCE CRITERIA

Cost Structure

Our experience with industry and U.S. government agencies shows that few companies or logistics bases are comfortable with setting values on shortage or order costs. In the government, service levels are preferred as a criterion. The ratio per unit of setup cost to holding cost is then the key variable. In this study, these values were based on the mean demand per period as in Berry (1972), Callarman and Hamrin (1979), and Wemmerlov and Whybark (1984). They were established by setting the time between orders (the TBO in number of periods of demand) at 2, 4, 6, 8, and 10. Using the mean demand per period, the a/h (setup to holding) ratio which would make each corresponding TBO optimal was determined and incorporated in each model. These values also correspond to our experience with industrial concerns.

Several measures may be applied to evaluate the performance of lot sizing approaches relative to service level. These measures include the number of stockouts and the shortages as a percentage of demand for the short period. In addition, a relative cost measure was obtained by comparing standard inventory cost for the algorithm with the cost determined by the optimal Wagner-Whitin algorithm.

Relative Cost

Previous studies have used the Wagner-Whitin heuristic as the baseline for cost comparisons. Unfortunately, the W-W procedure is suboptimal in the case of a rolling horizon and probabilistic demand. Arguments for the use of the Wagner-Whitin heuristic as the baseline are:

1. Wagner-Whitin is the baseline in the deterministic case.
2. It is not known before hand which rule will outperform the others.
3. Use of the Wagner-Whitin heuristic provides an easy comparison to previous work.

We feel these reasons do not justify the use of a single heuristic as a basis of comparison. It is true that we do not know what the optimal inventory cost of a probabilistic lot size problem will be until the demands have already been satisfied, i.e., we do not know what our future demands will be. However, by comparing the cost obtained through the use of a heuristic (when demand is considered stochastic) with the optimal cost obtained by Wagner-Whitin over the entire demand "history" (when considered deterministic), we obtain a true, fixed reference for comparision.

The key to this fixed reference is in the interpretation of the cost comparison. Specifically, the difference in cost between the optimal and the hueristic may be thought of as the maximum amount of money we would be willing to pay for perfect knowledge of our future demand (referred to as the expected value of perfect information or EVPI). (See Raiffa, 1968.)

Number of Stockouts

Wemmerlov and Whybark (1984), Tsado (1985a), and others arbitrarily set service levels in order to handle the question of stockouts. By service level, we mean there exists enough safety stock to assure demands are met a specified percentage of the time. Generally, levels between 90 and 99.999 percent are chosen. As a result, the stockout question is largely ignored.

Since we assume that stockouts have a "variable" cost (i.e., the cost of a stockout to one organization may be quite less than that perceived by another), setting an arbitrary service level may not be appropriate. Further, by pre-determining a service level, the effects of a lot size algorithm on inventory (holding and setup) costs and stockout costs may be confounded by the safety stock issue.

In a manner similar to that employed by Bookbinder and H'ng (1986), we chose to "count" the number of times a lot size heuristic produced a stockout. Obviously, this number will vary according to TBO level. Therefore, we chose to compute the stockout "cost" as the number of times a stockout occurred expressed as a percentage of the number of replenishments made. For example, given a 52-period demand "history" with a TBO level of 2, 5 stockouts out of 26 replenishments (approximately) would yield a stockout "cost" of 0.1923, i.e., about 19.23% of the replenishments made would experience a stockout. For a TBO of 6 periods, 5 stockouts would imply a "cost" of 57.69%.

Percentage Short per Stockout

Another factor in the stockout question is the amount of shortage when a stockout occurs. The average shortage per stockout is therefore an important "cost" consideration. However, an average shortage of N units does not provide a significant amount of information. There are two ways of handling this problem. One is to express the shortage as a percentage of average demand. Another approach is to express the shortage as a percentage of the actual demand for the period in which a shortage occurred. We chose the latter. Justification for our selection follows.

Consider an average demand of 500 units. If we forecast a demand of 550 units where the actual demand is 600 units, then our percentage short is only 8.3% of actual demand. If we had used average demand, we would have shown a shortage of 10%. Now assume an average demand of 50 units, a forecasted demand of 100 units ,and an actual demand of 150 units. We show a shortage of 33.3% of actual demand rather than a misleading 100% of average demand. In both cases, the forecast was 50 units greater than the average demand, and the actual demand was 50 units greater than the forecasted demand. Obviously, shortage "cost" expressed as a percentage of actual demand is a more accurate estimate of the true "cost" associated with a shortage.

PROCEDURE

Assumptions

The assumptions used to develop the single-stage, production lot size problem are similar to those employed by other researchers and are as follows:

1. Demand is probabilistic and is forecast using a limited amount of prior history.
2. A fixed cost is incurred for each setup.
3. The inventory holding cost is a function of the amount of inventory on hand at the end of a given period.
4. Production lead time is zero (i.e., we have enough inventory at the end of a production period to meet that period's demand).
5. All demands are met at the end of each period (i.e.,the order for the next period will cover shortages for the current period.
6. There is no safety stock except that which is inherent to a particular lot size heuristic.
7. Back orders are allowed.
8. There is no monetary penalty for shortages in the cost calculations (i.e., shortages are handled as a separate criterion.
9. Demand for the next period is not known with certainty.
10. An updated forecast is available for any period.

General Procedure

Research on lot size procedures has been performed by Silver (1978), Askin (1981), Bookbinder and H'ng (1986), and Bookbinder and Tan (1988). Our procedure (Figure 1), while developed prior to our knowledge of the previous works, is similar to that suggested by Bookbinder and Tan as follows:

1. Use a focus forecast from simple exponential smoothing and exponential smoothing with trend models for demands over the rolling horizon.
2. Treat the forecast demand as deterministic and employ a specific lot size heuristic.
3. If on-hand inventory is positive, the production quantity will be the amount obtained from the lot size heuristic minus the on-hand inventory.
4. If on-hand inventory is negative (i.e., a stockout has occurred), the production quantity will be the amount obtained from the lot size heuristic plus the amount backordered.
5. For each period, compare the on-hand inventory to the forecast for the next period. If the forecast exceeds the inventory position, schedule a setup for the next period, otherwise continue.
6. When the next period's demand is realized, demand is met or a shortage occurs. If short, schedule a setup for the next period, otherwise look at the next period's forecast (Step 5).
7. Develop an extended forecast only when a setup is scheduled.
8. Continue this procedure until all available demand data is exhausted.
9. Discount the inventory holding cost for all on-hand inventory used to satisfy demand beyond the last period in the data set.

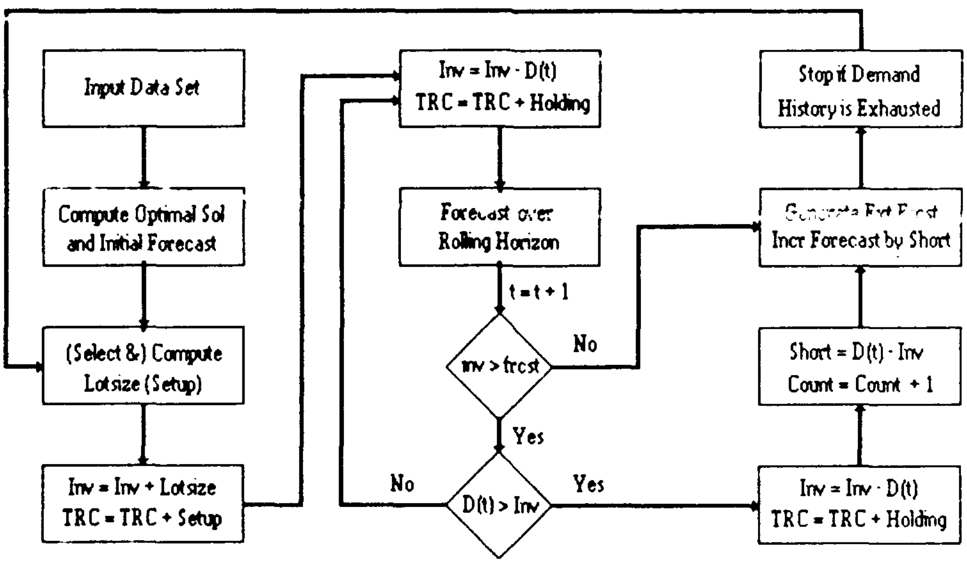

Figure 1. Flow Chart of General Procedure

Although this study used actual demand data rather than a simulation, computer software was developed to generate the forecasts, compute production policies via the various lot size heuristics (and the optimal Wagner-Whitin cost), and to compute the criterion measures associated with each technique. The forecasting procedure and the lot size procedures were programmed in MICROSOFT QuickBASIC and run on an IBM XT compatable microcomputer.

Forecast Procedure

The complete forecast was generated over the entire demand history of each data set (on a rolling horizon basis) prior to implementation of any lot size procedure. Estimates of level demand and trends (when Holt's exponential smoothing model was used) were stored in memory. Although the forecast for each period was used in the lot size procedure, extended forecasts were only developed when required by the particular lot size heuristic employed. To provide a compact computer algorithm, the simple exponential smoothing procedure was incorporated into Holt's procedure by setting the trend parameter, *g*, equal to zero.

"Focusing", i.e., selecting the "best" forecast model, was carried out by keeping track of the mean absolute deviation and the smoothed error tracking signal (bias) for each individual or single forecast. Estimates of the MAD were also exponentially smoothed. The forecast with the best current MAD was selected for the focused model if the bias was within acceptable limits, specifically between -0.8 and 0.3.

Silver and Peterson (1985) argue that a negatively biased forecast (where forecast exceeds the demand) is preferable to a positively biased forecast (where demand exceeds the forecast) since being a few items overstocked is preferable to consistently being short (causing too many premature setups). Wemmerlov and Whybark (1984) specifically avoid the use of biased forecasts by adjusting the average actual demand per period to equal the average forecasted demand per period. While easily done for simulated demand data, this is generally not appropriate for empirical demand forecasted on a rolling horizon basis. Research by Lee, Adam, and Ebert (1987) shows that "bias is the only measure that satisfactorily reflects inventory carrying costs...(and) only bias displays any reasonable association with the shortage cost and shortage units..."

Since carrying cost is caused by over forecasting (what Silver and Peterson (1985) referred to as positive bias) and shortage costs are caused by under forecasting (negative bias), the use of an unbiased forecast (as used by Wemmerlov and Whybark, 1984) might seem reasonable. The research by Lee et al. (1987), however, shows that "the structures of these two component costs may not be symmetrical about the zero bias level." Unfortunately, they do not provide guidelines as to what the nominal bias levels may be.

The specific bias levels used in our forecast model were determined in conjunction with an outlier discounting criterion. Outliers were discounted by keeping track of the average demand and standard deviation of the series at each point in the forecast "cycle". If an outlier exceeded 4 standard deviations, the actual demand was reduced to the mean plus 4 standard deviations for forecast purposes. This provided a stabilizing influence on the forecast which otherwise would have been provided by human intervention. On the downside, the forecast model would lag slightly behind a true shift in the mean of the demand series. This type of lag, however, is a standard "penalty" for exponentially smoothed forecast procedures.

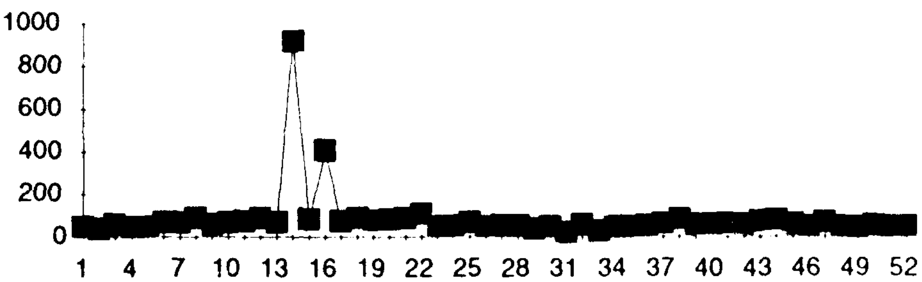

Figure 2. Example of Level Demand Pattern

As an example, the data set in Figure 2 exhibited a steep downward trend in the forecast model due to large upward spikes (outliers) relatively early in the demand series. The steep downward trend was leveled somewhat by discounting the outliers and then varying the bias criteria in an effort to eliminate a large series of zero forecasts caused by the apparant trend.

EVALUATION METHODOLOGY

Demand Data Sets

Of 500 original sets of demand data that were obtained from a national product distribution company, 207 were usable in that no zeros or alphabetic characters appeared as the monthly demand. The demand was based on 52 weekly periods and was clasifed by difficulty of forecasting and by variability as measured by the coefficient of variation (CV). Demand patterns were classified as easy to forecast (level demand, trending, or gradual change to trending) or difficult to forecast (abrupt change, or continuous up and down trending).

Table 1
Classification of Demand Data Sets

Pattern/CV	0 < CV ≤ .5	.5 < CV ≤ 1	CV > 1
Linear	12	6	5
Non-linear	8	5	0

The categories for CV were low (0 < CV ≤ .5), medium (.5 < CV ≤ 1.0) and high (CV > 1.0). These ranges have been observed frequently in several organizations. Table 1 shows the classification of demand patterns and the breakout of 36 data sets (Group 1) chosen randomly from the 207 usable data sets. Figures 2 (previous page), 3 and 4 are examples of Group 1 data.

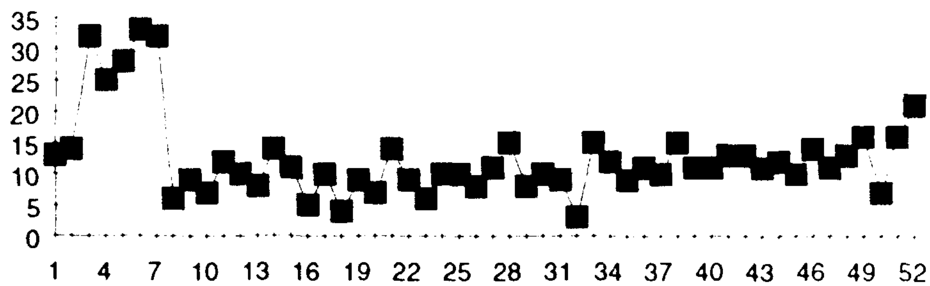

Figure 3. Example of Linear Trending Demand Pattern

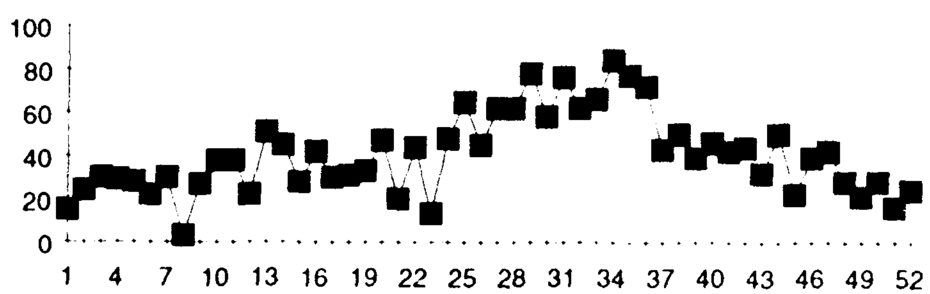

Figure 4. Example of Nonlinear Demand Pattern

Five other demand data sets from another company were also used. These data sets (Group 2) spanned 78 months but were not classified as to trending or variability. All data sets are available in Cline (1989).

Experimental Design

For the Group 1 data sets, a 5 x 5 x 2 x 3 (lot sizing technique x TBO x data set type x variability class) experimental design was used with repeated measures on the first two factors. Lot sizing technique (Eisenhut, EOQ, Silver-Meal, Tsado, and W-W Heuristic) and TBO (2, 4, 6, 8, and 10) were the primary factors of interest. Data set type (easy vs. difficult to forecast) and varibility class (low, medium, and high) served as stratification variables. For the Group 2 data sets, only lot sizing technique and TBO were included in the analysis. Due to the fewer number of Group 2 data sets, stratification was not feasible.

Each demand data set served as input to each of the five lot sizing techniques for each of the five TBO levels. Values for the three performance measures (inventory cost relative to the optimal W-W, stockouts as a percentage of replenishments, and shortage as a percentage of actual demand) were thus collected on 900 runs (36 data sets x 5 lot sizing techniques x 5 TBO levels).

RESULTS

Group 1 Data

The results collapsed across the 36 Group 1 data sets are summarized in Table 2 for the relative inventory cost, percentage of stockouts, and percentage short.

Table 2
Inventory Costs, Stockouts, and Shortages vs. Lot Sizing Technique and TBO

Technique/TBO	2	4	6	8	10	Average
	IC SO %S	IC SO %S	IC SO %S	IC SO %S	IC SO %S	IC SO %S
Eisenhut	38.4 11.6 27.6	29.6 12.7 23.1	36.9 14.1 17.7	46.0 15.2 18.3	74 7 16.4 30.9	45.1 14.0 23.5
EOQ	45.1 8.2 21.6	29.2 16.7 28.5	23.6 22.7 26.6	22.3 26.0 31.6	18.1 34.2 37.6	27.7 21.5 29.2
Silver-Meal	41.6 11.7 25.7	25.7 18.4 32.5	24.5 18.7 23.6	27.9 21.3 21.4	27.0 29.3 31.6	29.4 19.9 27.0
Tsado	39.6 10.9 28.2	23.1 21.0 29.3	19.7 26.2 32.4	18.4 29.9 31.8	16.3 34.9 42.4	23.4 24.6 32.8
W-W Heur	41.0 9.1 24.0	32.5 14.4 24.6	43.3 11.5 20.8	111.3 5.7 17.3	323.5 0.5 7.5	110.3 8.3 18.9
Averages	41.1 10.3 25.4	28.0 16.6 27.6	29.6 18.6 24.2	45.2 19.6 24.1	91.9 23.1 30.0	

IC - inventory cost relative to Wagner-Whitin
SO - stockouts as a percentage of replenishments
%S - shortage as a percentage of actual demand

As illustrated by Figures 5, 6 and 7 (next page), the various lot sizing techniques performed differently with respect to standard inventory cost (holding and setup) than they did with respect to shortage related measures. In terms of inventory costs (Figure 5), the EOQ, S-M and Tsado algorithms performed significantly better than the Eisenhut and W-W heuristics. Although a Tukey test with an alpha level of 0.05 demonstrated statistical significance, the difference was of practical importance only when the TBO was large (8 or 10 periods).

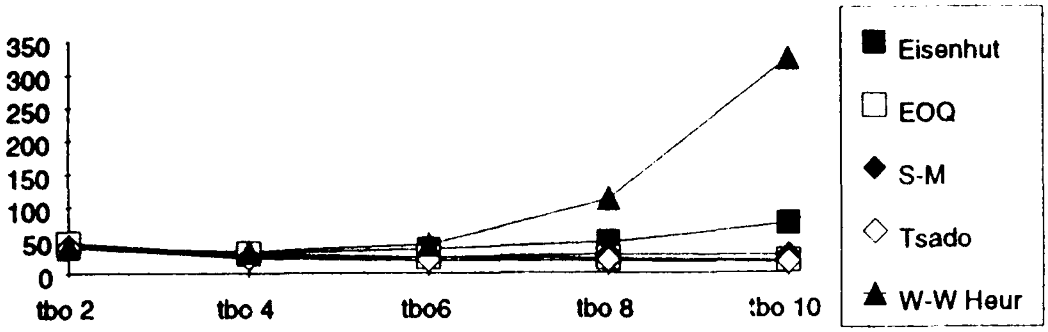

Figure 5. Relative Inventory Cost as a Function of Lot Sizing Technique and TBO

Conversely, Figures 6 and 7 show the Eisenhut and W-W heuristics resulted in significantly fewer shortages and a lower percentage of items short per stockout than the other three algorithms. The variability in the number of stockouts (in terms of percentage of replenishments) across the different levels of TBO was smallest for the Eisenhut PPB technique with means ranging from 12% to 18%. The W-W heuristic, while slightly less stable than the Eisenhut algorithm, was statistically the best overall performer for number of shortages (with an alpha level of 0.01). Variability in percent short across the TBO's was essentially the same for all five algorithms.

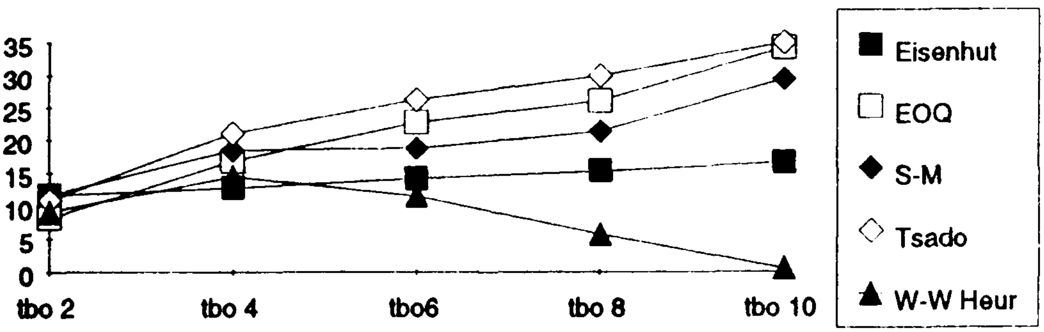

Figure 6. % Stockouts as a Function of Lot Sizing Technique and TBO

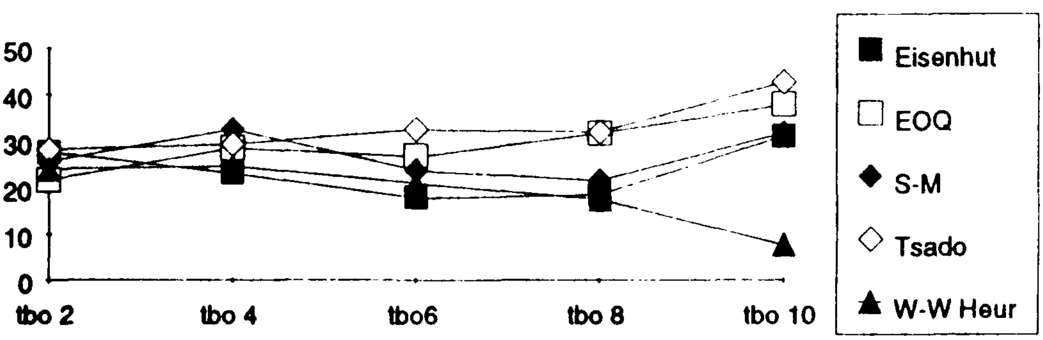

Figure 7. % Short as a Function of Lot Sizing Technique and TBO

In all cases, the W-W heuristic performed worst with respect to inventory cost but best with respect to shortages. At a TBO of 10, the W-W heuristic provided solutions with virtually no stockouts and less than 10% of actual demand short when a stockout did occur. The price for this performance, however, was an average inventory cost of more than three times the optimal solution. It appears that both the W-W heuristic and Eisenhut algorithms maintain a significant amount of inherent safety stock whereas the other algorithms tend to "run lean". Additional inventory increases the inventory holding costs but reduces the number of stockouts due to being a few items short.

Whether the demand pattern was easy or difficult to forecast did not significantly affect either the inventory costs or the number of shortages. However, difficult demand data did result in a significantly smaller percentage short per stockout due to "over forecasting" by the linear forecast model.

The level of variability in the data set (as indicated by the coefficient of variation) was a significant factor in that all three variability classes differed significantly on all criterion measures (with lower costs typically associated with lower variability as would be expected). The difference did not hold for a TBO of 2 where the inventory costs averaged 41.3, 41.7, and 39.0 for the low, medium, and high variability classes, respectively. At high variability for a TBO of 2, there tended to be stockouts which lowered holding costs.

Inventory cost performance as a function of TBO tended to validate the "rule of thumb" advocated by Wemmerlov and Whybark (1984) which states that the length of the rolling horizon should be approximately three times the average time between orders. A TBO of 4 provided the lowest inventory cost using a rolling horizon of 12 periods. However, performance as measured by the number of stockouts and amounts short per stockout did not support this recommendation. Wemmerlov and Whybark also advocate a rolling horizon of at least five times the average time between orde. , for the Wagner-Whitin heuristic since it uses the entire length of the rolling horizon to make its initial production decision. Recall, in the Wemmerlov study, that service levels were held constant for all factors.

Group 2 Data

With only five data sets, the simplified analysis examined only lot sizing technique and TBO level. The results paralleled those for the Group 1 data. Lot sizing technique and TBO were significant factors. The same techniques performed better for the same respective criteria, however, with fewer data sets, the level of significance was smaller. Additionaly, these data sets had long periods of level demand which also tended to obscure differences in the techniques.

Validation

Since the random sample was not balanced (i.e. non-equal replications in all cells), additional samples were drawn so that an equal number of data sets (n = 5) occurred in each cell. Interaction effects could then be more appropriately analyzed.

The results not only paralleled the previous study, they were much stronger and the conclusions they supported were more clearly delineated. The balanced design allowed an interaction analysis that supported an interpretation for practical application as follows:

1. The level of forecasting difficulty does not affect the choice of algorithm if focus forecasting is used.
2. The selected criterion does affect the choice of algorithm. Although not statistically different, the order of performance based on the selected criterion is given in Table 3.
3. Increasing the TBO magnifies the difference between the algorithms, i.e., at large TBO levels (8 and 10), the first -ranked algorithm is statistically superior.

Table 3
Top Performing Algorithms By Performance Criterion

Criterion	Best	Better	Good
Cost:	Tsado	Eisenhut PPB	Silver-Meal
% Short:	W-W Heuristic	Eisenhut PPB	
% Stockouts:	W-W Heuristic		

SUMMARY

Wemmerlov (1989) states that "lot sizing research is not dead". Our results indicate this is true. There is much to be gained by selecting the right lot sizing technique based on the desired criterion. If minimizing shortages is the criterion of choice, Eisenhut Part Period Balance or the Wagner-Whitin heuristic is clearly best. Otherwise, EOQ, Silver-Meal or Tsado are equally good choices. The big gap in research is testing lot sizing techniques using empirical demand data when resource constraints are present. Therefore, further research in this direction is warranted.

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